On the Interplay Between Resource-Management Overhead and Performance in Sensor Networks: An Information Theoretic Approach

Mahshid Rahnamay-Naeini[*‡], Nasir Ghani[†], and Majeed M. Hayat[*‡]
E-Mail: mrahnama@ece.unm.edu, nghani@usf.edu, hayat@ece.unm.edu
[*]Department of Electrical and Computer Engineering, University of New Mexico, Albuquerque, NM, USA
[†]Department of Electrical Engineering, University of South Florida, Tampa, FL, USA
[‡] Center for High Technology Materials, University of New Mexico, Albuquerque, NM, USA

***Abstract*—Resource management (RM) is a critical action in systems with limited resources such as sensor networks. Efficient RM depends heavily upon the availability of accurate information on the state of available resources. However, exchange of state information incurs an overhead on the system. In this paper, the sensing and computing resource management in sensor networks is considered. A lossless, distributed source-coding framework is presented to model the exchange of state information. The framework enables the characterization of the interplay between the performance and overhead of RM by leveraging the correlation among the state information of various nodes. Moreover, the proposed framework enables an improved estimate of the lower bound for the minimum control overhead necessary to accurately describe the state of nodes in the network. This improvement is achieved by exploiting the correlation among the state information of nodes as well as the available information on prior resource allocation actions as side information.**

***Index Terms*—Sensor networks, resource management, distributed computing, control overhead, information theory, distributed source coding, side information.**

I. Introduction

Resource management (RM) is a critical action in systems with limited resources such as sensor networks. In this paper, we focus on the computing and sensing resource-management problem in sensor networks. Managing sensing and computing resources is important in many sensor-network applications such as pervasive computing, mobile computing, battlefield surveillance, control and situational awareness, and target tracking [1]–[3]. In such applications local agents (users and sensor nodes) in the field as well as remote users submit sensing and computing requests to the system. Sensor networks respond to these requests by means of collaborative processing while using distributed sensing and computing resources. In this paper, resource allocation refers to assigning tasks associated with requests to sensor nodes. Hence, we use the term *task assignment* (TA) and *resource allocation* interchangeably in this paper. Furthermore, the proposed analytical framework in this paper can be applied to any arbitrary resource management problem in distributed systems such as dynamic wavelength assignment in optical networks and computing resource management in cloud systems.

Efficient RM depends heavily upon the availability of accurate resource-utilization information, which we term *state information* [3]–[6]. However, the inevitable exchange of state information consumes valuable energy and communication resources and incurs a cost in the system termed RM *control overhead*. Recently, information theory has been used to characterize the control overhead of networks [7]–[10], where the control overhead is thought of as the rate of information exchange. In such works, point-to-point rate-distortion theory [11] has been utilized to capture the trade-off between control overhead and the distortion in the state information (consequently the performance of a given protocol).

Different from the rate-distortion approach, here we present a *lossless*, distributed source-coding framework to characterize the interplay between RM performance and the overhead it incurs in sensor networks. This has been done by taking into account the correlation among the state information of various nodes while observing that an RM protocol can affect the correlation among the state information of nodes. Notably, our framework ties each RM protocol to an overhead. This is because in a lossless distributed source-coding framework the correlation among the state information specifies the rate region for accurate communication, determined by constraints on the rate of each node and sum of the rate of nodes. We also derive an improved lower bound for the rate region (associated with an arbitrary RM protocol) by exploiting the correlation among the state information of nodes as well as knowledge of prior resource-allocation actions as side information.

We emphasize that the interplay addressed by rate-distortion–based formulations is different from that addressed by our framework. Rate-distortion–based formulations characterize the trade-off between the rate of information and distortion by considering a class of codes that satisfies the rate and distortion constraints. On the other hand, our framework characterizes the interplay between performance of RM and its overhead by means of a *class of protocols*, which each satisfies certain performance and overhead while assuming sensor-network applications for which distortion in the state information is not tolerable. For example, a constraint on the control overhead will yield a class of admissible RM-protocols, from which we can extract the best achievable RM performance.

This paper is organized as follows. A brief overview of related work is presented in Section II. Section III defines the sensor network and resource management model. In Section IV, a distributed source-coding framework for exchanging the state information of nodes is formulated and its rate region is characterized. Numerical results for an example sensor network and discussion on the interplay between the control overhead and RM performance are presented in Section V. Finally, concluding remarks are presented in Section VI.

II. Related Work

Recently, information theory has been adopted for characterizing the control overhead of networks. The pioneering work presented by Gallager [12] is one of the earliest contributions that used information theory in characterizing the network overhead for tracking source and receiver addresses. In [7] and [8] the minimum overhead of maintaining state information (link state and motion state, respectively) to be used in routing protocols across a mobile ad hoc network is formulated as a rate-distortion problem. The assumption in [7] and [8] is that the state information associated with various nodes/links are mutually independent; hence, the rate-distortion formulation is considered for a single component. The authors of [10] use rate-distortion theory to investigate the optimal timing for updating the bandwidth information of the links. In [9], the relation between network performance and information rate is captured by extending the definition of distortion measure to capture network performance.

It is to be noted that all the aforementioned works consider point-to-point information theory in characterizing the interplay between network overhead and distortion. Network information theory [11], on the other hand, provides strong tools for characterizing the information exchange in a distributed fashion when there is correlation among the state information of different sources. However, to the best of our knowledge networked information theory has not been used heretofore in investigating the control overhead of networks. While distributed source coding theory has been widely used in sensor networks in the last decade, its utility has been limited to the context of distributed sensing [13]. Although tracking the state of nodes in a network can be viewed as a distributed-sensing problem, the signal of interest here is tied to the network characteristics, protocols and policies, which can collectively define the performance of the network. Here, the distributed source coding model may warrant extension or modifications to capture the information characteristics and their availability in various part of the network. In this paper we use the generality of distributed source coding theory to characterize the interplay between performance and the control overhead of RM in sensor networks through the correlation among state information of various nodes.

III. System model

In this section, we model sensor networks and resource management. The notation and terminology used in this paper are summarized in Table I. Consider a sensor network consisting of clusters of sensors as shown in Fig. 1. Each cluster has a control node (CN) that is responsible for monitoring the resource utilization, maintaining the state information of nodes and performing resource allocation. Here, the problem of investigating the control overhead of sensor networks is broken down to the same problem for individual clusters of nodes. Hence, we will focus on one cluster hereafter with the understanding that the same procedure will be applied to all clusters of the network.

TABLE I
Table of notations

N	$\triangleq$	Number of nodes in the cluster
$Q_i(t)$	$\triangleq$	Stochastic queue size of node i at time t
t_U	$\triangleq$	Update instance in which nodes send state information to CN
t_S	$\triangleq$	Sampling instance in which nodes take samples of their $Q_i(t)$
t_A	$\triangleq$	Task assignment instance at which CN assigns tasks to sensors
n	$\triangleq$	Number of t_S and t_A instances in a $[t_{U_{i-1}}, t_{U_i}]$
M_j	$\triangleq$	Random number of tasks arrived to CN during $[t_{A_{j-1}}, t_{A_j}]$
Y_{ij}	$\triangleq$	Random number of tasks assigned by CN to node i at t_{A_j}
Y_i	$\triangleq$	The random variable associated to node i, which we shall view Y_{ij}s as i.i.d samples of it
P_i	$\triangleq$	The random variable associated to node i, which its realizations P_{ij}, represents the number of tasks being processed by node i during the interval $[t_{S_{j-1}}, t_{S_j}]$
L_{ij}^k	$\triangleq$	Random number of tasks assigned to node i by local agent k, during the interval $[t_{S_{j-1}}, t_{S_j}]$
L_{ij}	$\triangleq$	$\sum_K L_{ij}^k$, where K is the number of local agents in the cluster
L_i	$\triangleq$	The random variable associated to node i, which we shall view Y_{ij}s as i.i.d samples of it
U_i	$\triangleq$	The random variable associated to node i, which its realizations U_{ij}, represents the number of tasks assigned to node i by its internal system during the interval $[t_{S_{j-1}}, t_{S_j}]$
X_{ij}	$\triangleq$	Random jump size in the queue size of node i i.e., $\lvert Q_i(t_{S_j}) - Q_i(t_{S_{j-1}})\rvert$
X_i	$\triangleq$	The random variable associated to node i, which we shall view X_{ij}s as i.i.d samples of it
F_{TA}	$\triangleq$	Task assignment protocol/function
$W(F_{TA})$	$\triangleq$	Overhead of state information exchange associated with F_{TA}
$Z(F_{TA})$	$\triangleq$	Performance measure of F_{TA}
$\mathcal{C}(F_{TA})$	$\triangleq$	Correlation among X_is resulted by F_{TA}

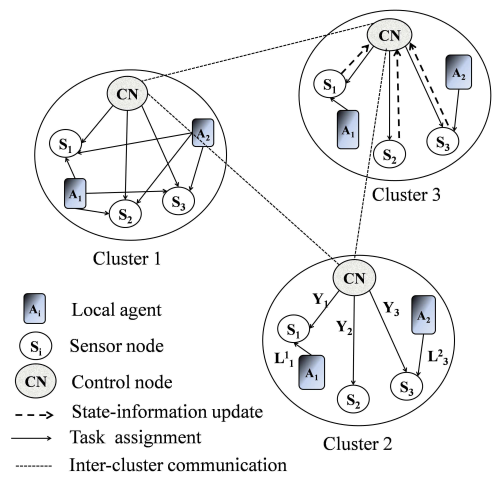


Fig. 1. A sensor system and its clusters.

A. Resource allocation and information exchange

The main role of sensor nodes in a cluster is to process tasks and send the state (resource utilization) information to the CN. We assume that the state information of nodes may be used by RM or other network protocols. We use the queue size of tasks (awaiting tasks) in a node to estimate the resource availability at the node. We term the intervals $[t_{U_{i-1}}, t_{U_i}]$, $[t_{S_{j-1}}, t_{S_j}]$, and $[t_{A_{j-1}}, t_{A_j}]$ as *update interval*, *sampling interval*, and *TA interval*, respectively, as shown in Fig. 2. We assume that each update interval consists of n sampling and TA intervals. Note that for convenience we assume equal number of sampling and TA instances in an update interval; however, these instances are not assumed to be synchronized.

A sensor node i encodes and then sends n samples of its queue size $Q_i(t)$, sampled at the instants t_{S_j}s, to the CN at the end of each update interval. We assume that the CN and its constituent nodes in a cluster communicate directly over noiseless links. We further assume that sensing and computing requests arrive at the CN at random times while each request is comprised a random number of tasks. The CN buffers tasks upon arrival and assigns them (total M_j tasks) to the nodes at t_{A_j} based upon a prescribed (static) RM protocol. We represent the way that CN assigns tasks to sensor nodes by a function F_{TA}, which may follow different objectives such as load balancing and minimizing energy consumption of the network. More specifically, we write $F_{TA}(M_j) = \mathbf{Y}(t_{A_j})$, where $\mathbf{Y}(t_{A_j}) = (Y_{1t_{A_j}}, Y_{2t_{A_j}}, \ldots, Y_{Nt_{A_j}})$ with the constraint $\sum_{i=1}^{N} Y_{it_{A_j}} = M_j$. Refer to Table I for the definition of notations. Here, the Y_{ij}s are independent and identically distributed (i.i.d) across time (i.e., for $j = 1, \ldots, n$).

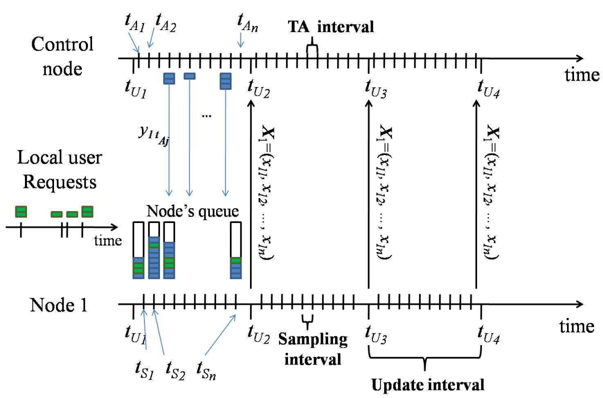


Fig. 2. Update, sampling and TA intervals.

B. Dynamics of queue sizes

The dynamics of the queue size of a node is governed by the random variables associated with the number of tasks being processed at the node, tasks being generated in the internal system of the node (by the node's operating system or software agents), tasks being received from local agents, and tasks being assigned to the node by the CN. Note that samples of queue sizes, $Q_i(t)$, at instants t_{S_j} for $j = 1, .., n$ are correlated due to the dependency of the queue size at any sampling time on the number of tasks queued at the node at previous sampling times. However, jumps in the queue size are independent in time because all the random variables affecting the queue size can be assumed to be independent in time (i.e., instants t_{S_j} for $j = 1, .., n$). More precisely, we represent the dynamics of the queue size by jumps in the queue size as

$$X_i = Y_i + U_i + L_i - P_i. \quad (1)$$

C. Correlation among random variables

As depicted for a generic random variable in Fig. 3, the random variables affecting the queue size of a node according to (1) are independent in time but they may be correlated across sensor nodes (except for U_i and P_i, which are independent across nodes because they are node specific). Generally, the Y_is of different nodes can be correlated due to

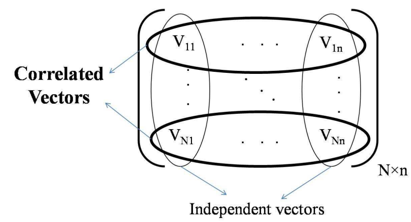


Fig. 3. A random matrix for a generic random variable.

the adoption of certain F_{TA} by the CN. We will consider the following examples of F_{TA}, which will illustrate the correlation among the Y_is of different nodes due to the system adopting a resource-allocation protocol, F_{TA}, say. First consider the protocol F_{TA_1} that randomly distributes tasks among sensor nodes based upon a uniform probability mass function. Clearly, it is expected in this case that the Y_is exhibit minimum correlation; note however that correlation may still exist due to the constraint $\sum_{i=1}^{N} Y_{ij} = M_j$. In the second example, F_{TA_2} evenly distributes tasks among nodes, which results in a high degree of correlation among the Y_is. Other in-between scenarios can be associated with the F_{TA}s that exploit the characteristics of the nodes and the state information in implementing resource allocation.

Similarly, the L_is, for $i = 1, \ldots, N$, may be correlated due to task assignment protocol of local agents in the cluster (the way local agents directly assign tasks to the nodes). As the first example of task assignment protocol, let us assume that agents assign tasks to their closest node as depicted in cluster 2 of the network shown in Fig. 1. In this scenario, we assume nodes and agents are not mobile; as such, we can assume that each agent is associated to a specific sensor node (e.g., the closest) and will only submit tasks to that node. In this case, the correlation among L_is is minimum because of the random and independent task assignment of agents. In the next example, we assume that agents evenly distribute their tasks among all the nodes in the cluster as has been shown in cluster 1 in Fig. 1. In this case, there is a higher degree of correlation among the L_is than that in the previous example. Once again, in-between scenarios can exist for example when users assign tasks to their closest node while they are mobile and the closest node change in time.

Now based upon the correlation among the random variables affecting the queue size of a node, we can conclude that X_is of different nodes (for $i = 1, \ldots, N$) can also be correlated. We represent the correlation among X_is by $\mathcal{C}(\mathcal{F_{TA}})$, which as discussed earlier, depends upon F_{TA}, the task-assignment protocol of agents and the RM policy.

IV. Information-Theoretic Framework

A. Distributed Source Coding Model

Assume that at each update instant node i uses its encoder to encode $X_i^n = (X_{i1}, X_{i2}, \ldots, X_{in})$ separately from other nodes and then sends it to the CN. We term the information about the prior resource-allocation actions, Y_is, as *side* *information*. We assume the statistical characteristics of X_i and Y_i are available, as discussed in Section III. The CN uses its decoder to decode and reconstruct the state of nodes using the correlation among X_is and the side information Y_is. Specifically, the reconstructed state of node i is denoted by $\hat{X}_i^n = (\hat{X}_{i1}, \hat{X}_{i2}, \ldots, \hat{X}_{in})$ and the probability of error is $P_e^{(n)} \triangleq \mathsf{P}\{(\hat{X}_1^n, \ldots, \hat{X}_N^n) \neq (X_1^n, \ldots, X_N^n)\}$. Figure 4 illustrates this formulation schematically for a cluster with N nodes. Our formulation is an extension of Slepian-Wolf Theorem [14] to distributed lossless source coding with multiple side information.

An N-tuple $(R_1, \ldots, R_N)$ is said to be *achievable* for distributed lossless source coding if there exists a sequence of codes with these rates such that $\lim_{n\to\infty} P_e^{(n)} = 0$. Note that the control overhead of a specific RM protocol is defined as

$$W(F_{TA}) \triangleq \sum_i R_i. \tag{2}$$

In our framework, protocol F_{TA} leads to certain level of correlation among the state information, $\mathcal{C}(F_{TA})$, which subsequently specifies the overhead $W(F_{TA})$ based upon the distributed source coding model. Therefore, the performance $Z(F_{TA})$ and overhead $W(F_{TA})$ are tied together through $\mathcal{C}(F_{TA})$, which enables the characterization of the interplay between performance and overhead of the RM.

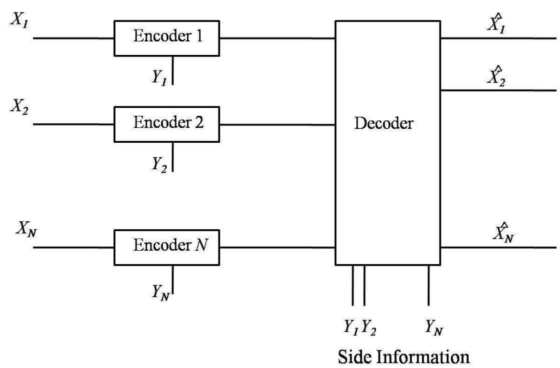


Fig. 4. The proposed distributed source coding model with side information to characterize the control overhead.

For the ease of reference and discussion of the theory, we define:

Formulation 1. A distributed source coding problem that does not use Y_is and nor does it use the correlation among X_is. This formulation is equivalent to N separate Shannon's lossless source coding problems.

Formulation 2. A distributed source coding problem that uses Y_is but it does not use the correlation among X_is. This formulation is equivalent to N conditional lossless source coding problems (single source coding problem with side information at the encoder and the decoder [11]).

Formulation 3. A distributed source coding problem that uses the correlation among X_is but it does not use Y_is. This formulation is a Slepian-Wolf problem.

Formulation 4. A distributed source coding problem that uses both Y_is and the correlation among X_is. This is our formulation in this paper.

Note that Formulations 1 and 2 fall in the area of point-to-point information theory.

B. Characterizing the rate region

We begin the characterization of the rate region of Formulation 4 by considering the distributed source coding model for a cluster with two nodes. The achievable rate region of the formulations mentioned in the last section has been depicted in Fig. 5. Note that in Fig. 5, we have shown the rate region of Formulations 2 and 3 in the special case when $H(X_1|X_2) < H(X_1|Y_1)$ and $H(X_2|X_1) < H(X_2|Y_2)$, where $H(.|.)$ represents the conditional entropy. Characterization of the achievable rate region of Formulation 3 has been presented by Slepian and Wolf in [14]. The achievable rate region of Formulation 2 is also known [11]. Characterizing the

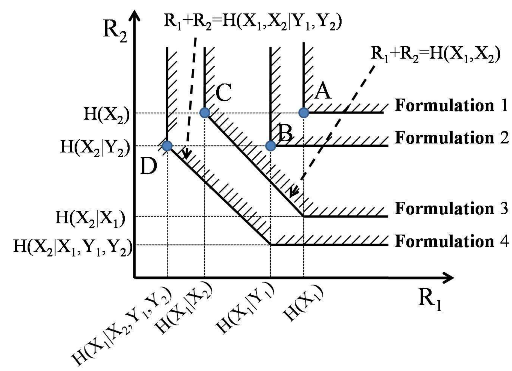


Fig. 5. The achievable rate region of the four formulations.

achievable rate region of the Formulation 4 is straight forward and can be explained as follows. Consider a special case in which there is only one encoder that jointly encodes the state information of the two nodes while the CN decodes the code for both sources using side information. In this case the outer bound of the *sum-rate*, defined as $R_1 + R_2$, for Formulation 4 can be written as $R_1 + R_2 \geq H(X_1, X_2|Y_1, Y_2)$. In general, however, this bound may not be achievable since nodes are encoding the sources separately (this is why it is termed outer bound). Now consider the case when nodes encode the sources separately while each node having access to the other node's state information. In this case, the rate of a node should satisfy $R_1 \geq H(X_1|X_2, Y_1, Y_2)$ based on Formulation 2. Combining these bounds results in the outer bound for the optimal rate region of Formulation 4, which can easily be shown to be achievable and tight (with a proof similar to that of the Slepian Wolf Theorem [15]) and therefore improves the lower bound on the minimum information rates.

Theorem 1. The optimal rate region for the problem in Formulation 4 with two nodes is

$$R_1 \geq H(X_1|X_2, Y_1, Y_2),$$

$$R_2 \geq H(X_2|X_1, Y_1, Y_2) \text{ and}$$

$$R_1 + R_2 \geq H(X_1, X_2|Y_1, Y_2).$$

Theorem 1 can be extended to an arbitrary number of nodes in the cluster as follows.

Theorem 2. Let $\mathcal{S} \subset \{1, 2, ..., N\}$. The optimal rate region for the problem in Formulation 4 with N nodes is

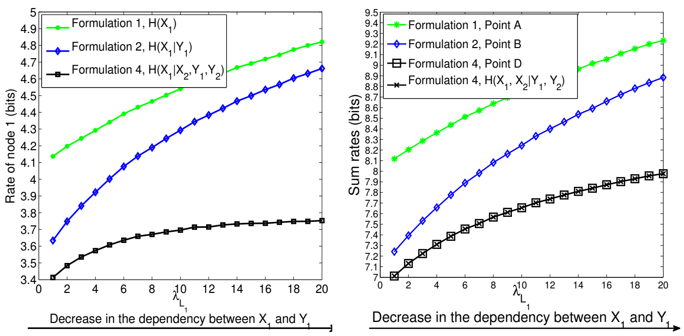
$$\sum_{j\in\mathcal{S}} R_j \geq H\big(X(\mathcal{S})|X(\mathcal{S}^c), Y(\mathcal{S} \cup \mathcal{S}^c))\big),$$

where $X(\mathcal{S})$ is the set of X_is for $i \in S$ and $X(\mathcal{S}^c)$ and

$Y(\mathcal{S} \cup \mathcal{S}^c)$ are defined likewise. Note that in this special case of distributed source coding problem the rate region of the framework depicted in Fig. 4 would be the same as the one stated by Theorem 1 and 2 even if the side information Y_is were only available at the decoder. However, in lossy distributed source-coding problems these two cases would result in different rate regions. The next point to note here is that the lower bounds derived based on these approaches, are asymptotically achievable (as the number of samples in an update interval goes to infinity, i.e., $n \to \infty$); hence, they provide bounds on the overhead of the network [7]–[10].

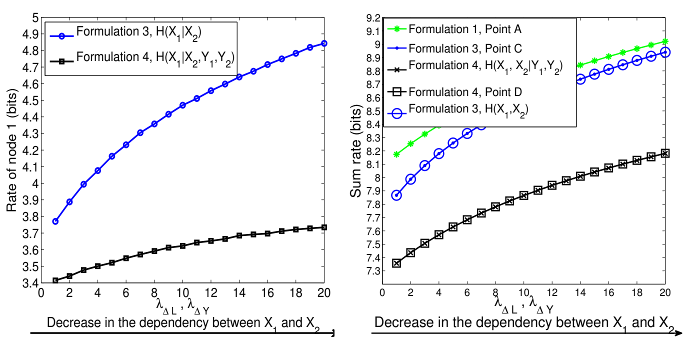
V. Numerical Evaluation

Theorems 1 and 2 provide analytical expressions of the lower bound for the minimum state information rate of nodes in a cluster. In this section, we consider a specific example of a sensor network and provide the numerical results calculated for the minimum control overhead of RM. For simplicity, we consider a cluster with two nodes.



(a) (b)

Fig. 6. (a) State information rate of node 1, and (b) total control overhead (sum of the rates of nodes) as a function of the dependency between X_1 and Y_1.



(a) (b)

Fig. 7. (a) State information rate of node 1, and (b) total control overhead (sum of the rates of nodes) as a function of the dependency between X_1 and X_2.

A. Settings of the example

We indicated in Section III that the Y_is of different nodes may be correlated due to F_{TA}. To model the dependency between Y_1 and Y_2 in our example we simply use $Y_1 = Y_2 + \Delta Y$. In this model F_{TA} affects the correlation between Y_1 and Y_2 (and consequently the correlation between X_1 and X_2, i.e., $\mathcal{C}(\mathcal{F}_{\mathcal{TA}})$) through ΔY, which represents the number of excess tasks assigned to node 1 compared to node 2. In a similar way, the correlation between L_is can be modeled by $L_1 = L_2 + \Delta L$. Here, ΔL depends on the task assignment protocol of local agents. Note that the correlation among L_is also affect the correlation among X_is. Therefore, F_{TA} and task assignment protocol of local agents affect $\mathcal{C}(\mathcal{F}_{\mathcal{TA}})$ through Y_is and L_is. In our example, we assume independent Poisson distributions for random variables with random-variable-specific Poisson parameters. With the above preliminaries and the dynamics of the queue size described in (1), we write X_1 and X_2 as

$$\begin{aligned} X_1 &= Y_2 + \Delta Y + U_1 + L_2 + \Delta L - P_1, \\ X_2 &= Y_2 + U_2 + L_2 - P_2. \end{aligned} \tag{3}$$

In our calculations, we have assumed $\lambda_{Y_2} = 6$, $\lambda_{L_2} = 3$, $\lambda_{U_1} = \lambda_{U_2} = 1$, $\lambda_{P_1} = \lambda_{P_2} = 4$, and $\lambda_{\Delta L} = \lambda_{\Delta Y} = 1$. Since all the random variables have Poisson distributions, we can numerically calculate the entropy, joint entropy and conditional entropy for different combination of the presented random variables. Moreover, we use the fact that sum of two independent Poisson random variables is a Poisson random variable and the difference of two independent Poisson random variables with parameters λ_1 and λ_2 has Skellam distribution,

$$p(k; \lambda_1, \lambda_2) = e^{-(\lambda_1+\lambda_2)} (\lambda_1/\lambda_2)^{(k/2)} I_{|k|}(2\sqrt{\lambda_1 \lambda_2}),$$

where $I_k(z)$ is the modified Bessel function of the first kind. The assumption on the Poisson distribution is commonly adopted in literature [16], [17] for the random number of tasks.

B. Numerical results

According to (1), X_i and Y_i may be correlated and the level of correlation between them is affected by the dynamics of the network and the node through U_i, P_i, and L_i. In general, as the randomness in these variables increases the correlation between X_i and Y_i decreases and rate of state information increases. In the extreme case, for which events in the system are deterministic (e.g., deterministic task processing time) the X_i values can be calculated based upon the Y_i values at the CN, and therefore, there is no need to exchange the state information and the control overhead will be zero. In our example, when U_i, P_i and L_i have high level of randomness (large λ parameter) then the correlation between X_i and Y_i decreases. Figure 6-a depicts the minimum state-information rate of node 1 as a function of λ_{L_1}, for Formulations 1, 2, and 4 calculated at points A, B, and D shown in Fig. 5, respectively. In our calculations, we have fixed the Poisson parameter of all the random variables in the system and only changed λ_{L_1}. From Fig. 6-a, we observe that as the dependency between X_1 and Y_1 decreases the rate of node 1 increases. The sum of the rates of nodes (total control overhead) for the formulations are calculated based on the sum-rate constraint in Theorem 1 and shown in Fig. 6-b. Figure 6-b also shows that the sum of the pair of rates shown in Fig. 6-a equals to the sum-rate constraint value calculated based on Theorem 1. This is due to the special position of the selected pair of rates (in Fig. 5).

We next investigate the effect of correlation between X_1 and X_2 on the control overhead. To see this effect in our example, we fixed the Poisson parameter of all the random variables and only changed $\lambda_{\Delta Y}$ and $\lambda_{\Delta L}$ (we assume $\lambda_{\Delta Y} = \lambda_{\Delta L}$). The minimum state-information rate of node 1 for Formulation 3 (calculated at point C in Fig. 5) and Formulation 4 are represented in Fig. 7-a. The minimum total control overhead for Formulation 3 and 4 are shown in Fig. 7-b. Notably Formulation 4 provides the smallest lower bound among other formulations for the minimum information rate of the nodes.

C. Interplay between performance and overhead

Recall that resource allocation protocols may affect the correlation between X_1 and X_2. In the results shown in Fig. 7, each $\lambda_{\Delta Y}$ value on x-axis can be interpreted as the representor of the F_{TA}s, which yields $Y_1 = Y_2 + \Delta Y$ with ΔY following a Poisson distribution with parameter $\lambda_{\Delta Y}$. The same can be said for $\lambda_{\Delta L}$ and the task assignment protocols of local agents. Note that each $\lambda_{\Delta Y}$ on x-axis can be mapped to a $\mathcal{C}(F_{TA})$ value. Similarly, the y-axis represents the control overhead of F_{TA}, i.e., $W(F_{TA})$. With this framework in place one can compare the lower bound on the minimum control overhead of various F_{TA}s analytically. Next. if the performance of F_{TA} is measured through a performance metric such as task completion time, energy utilization or life time of sensor network, then the interplay between $Z(F_{TA})$ and $W(F_{TA})$ can be characterized analytically.

To illustrate the interplay between the performance and control overhead of RM, consider the policies F_{TA_1} and F_{TA_2} introduced in Section III. The performance of F_{TA_1} in terms of task completion time and load balancing has been shown to be more efficient than that for F_{TA_2} in certain applications when the size of requests have a Poisson distribution [16], [17]. Therefore, in this scenario $Z(F_{TA_1}) > Z(F_{TA_2})$. Meanwhile, in this case $\mathcal{C}(F_{TA_1}) < \mathcal{C}(F_{TA_2})$ due to the small degree of correlation among the Y_is resulted from F_{TA_1}, which, in turn, implies $W(F_{TA_1}) > W(F_{TA_2})$. This example demonstrates a trade-off between performance and the control overhead of RM, which can be captured analytically through our framework. Similar statements can be made about the task assignment protocol of local agents. For example, the control overhead in the task-assignment protocol shown in cluster 2 of the network in Fig. 1 is larger than that of cluster 1 as a result of the different degree of correlation among the X_is. Hence, our framework paves the path to the characterization of the overhead-admissible RM protocols, from which we can extract the best achievable RM performance.

VI. Conclusions

Efficient resource management depends heavily upon the availability of accurate information on the state of available resources in the system. However, exchange of state information incurs an overhead on the network. In this paper, we formulated the exchange of state information in the network using a lossless, distributed source coding framework. We have shown that resource-management policies may lead to different levels of correlation among the state information of nodes. We then exploited the correlation among the state information of nodes to analytically characterize the interplay between the resource-management performance and overhead in a lossless source-coding framework. This framework provides an analytical tool to compare the performance and control overhead of various resource management policies. Moreover, we have derived an improved estimate of the lower bound for the minimum control overhead of RM by utilizing the correlation among the state information of nodes as well as the available information on prior resource-allocation actions as the side information.

Acknowledgment

This work was made possible by the Defense Threat Reduction Agency grant No. HDTRA1-13-1-0020 and contract No. DTRA01-03-D-0009-0026, and by NPRP 5 - 137 - 2 - 045 grant from the Qatar National Research Fund (a member of Qatar Foundation). The statements made herein are solely the responsibility of the authors.

References

[1] A. Pathak and V. Prasanna, "Energy-efficient task mapping for data-driven sensor network macroprogramming," *IEEE Transactions on Computers*, vol. 59, no. 7, pp. 955 –968, July 2010.

[2] T. P. Le, T. J. Norman, and W. Vasconcelos, "Adaptive negotiation in managing wireless sensor networks," in *Proc. of the 13th international conference on Principles and Practice of Multi-Agent Systems*, ser. PRIMA'10, 2012, pp. 121–136.

[3] V. Giordano, F. Lewis, P. Ballal, and B. Turchiano, "Supervisory control for task assignment and resource dispatching in mobile wireless sensor networks," *Cutting Edge Robotics*, pp. 133–152, 2005.

[4] J. Pezoa, S. Dhakal, and M. Hayat, "Maximizing service reliability in distributed computing systems with random node failures: Theory and implementation," *IEEE Transactions on Parallel and Distributed Systems*, vol. 21, no. 10, pp. 1531 –1544, oct. 2010.

[5] Z. Wang, M. Hayat, M. Rahnamay-Naeini, Y. Mostofi, and J. Pezoa, "Consensus-based estimation protocol for decentralized dynamic load balancing over partially connected networks," in *IEEE Conference on Decision and Control (CDC)*, Dec. 2011, pp. 4572 –4579.

[6] D. V. Pynadath and M. Tambe., "The communicative multiagent team decision problem: Analyzing teamwork theories and models,," *Journal of Articial Intelligence*, vol. 16, p. 389423, 2002.

[7] D. Wang and A. Abouzeid, "Link state routing overhead in mobile ad hoc networks: A rate-distortion formulation," in *The 27th Conference on Computer Communications IEEE INFOCOM*, April 2008.

[8] D. Wang and A. Abouzeid, "On the cost of knowledge of mobility in dynamic networks: An information-theoretic approach," *IEEE Transactions on Mobile Computing*, vol. 11, no. 6, pp. 995 –1006, June 2012.

[9] J. Hong and V.-K. Li, "Impact of information on network performance - an information-theoretic perspective," in *IEEE Global Telecommunications Conference GLOBECOM*, 2009, pp. 1–6.

[10] G. Cheng and N. Ansari, "Rate-distortion based link state update," *Computer Networks*, vol. 50, no. 17, pp. 3300–3314, December 2006.

[11] A. E. Gamal and Y.-H. Kim, *Network Information Theory*. Cambridge University Press, 2012.

[12] R. Gallager, "Basic limits on protocol information in data communication networks," *IEEE Transactions on Information Theory*, vol. 22, no. 4, pp. 385 – 398, July 1976.

[13] T. Berger, Z. Zhang, and H. Viswanathan, "The ceo problem [multiterminal source coding]," *IEEE Transactions on Information Theory*, vol. 42, no. 3, pp. 887 –902, May 1996.

[14] D. Slepian and J. Wolf, "Noiseless coding of correlated information sources," *IEEE Transactions on Information Theory*, vol. 19, no. 4, pp. 471 – 480, July 1973.

[15] T. Cover, "A proof of the data compression theorem of slepian and wolf for ergodic sources (corresp.)," *IEEE Transactions on Information Theory*, vol. 21, no. 2, pp. 226 – 228, Mar 1975.

[16] M. Bramson, Y. Lu, and B. Prabhakar, "Randomized load balancing with general service time distributions," *SIGMETRICS Perform. Eval. Rev.*, vol. 38, no. 1, pp. 275–286, Jun. 2010.

[17] M. Mitzenmacher, "On the analysis of randomized load balancing schemes," *Theory of Computing Systems*, vol. 32, pp. 361–386, 1999.